MU120 RBD

The Open University

Mathematics and Computing
A first level multidisciplinary course

Open Mathematics

RESOURCE BOOK D

Units 14 – 16

RB

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A first level multidisciplinary course

Open **Mathematics**

RESOURCE BOOK D

Units 14 – 16

Prepared by the course team

This is the last of the four resource books which are part of the MU120 course materials. It contains further practice questions for *Units 14–16*, and revision questions for the whole course, together with solutions.

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2.1

Contents

Unit 14 Space and shape

Question 1

In each case, decide whether or not the shapes given are similar.

(a) The body of an adult and that of a toddler of the same sex.

(b) Any two chessboards.

(c) A right-angled triangle whose sides adjacent to the right angle are 4 cm and 5 cm long, and another whose sides adjacent to the right angle are 12 cm and 15 cm long.

(d) The Earth and the Moon.

Question 2

(a) At any point on the Equator at midday (local time) on 21 March (the vernal, or spring, equinox), the Sun is directly overhead. In Manchester at midday on that day, the angle of elevation of the Sun is 36.5°. What is the latitude of Manchester?

(b) On 21 December in Manchester, the angle of elevation of the Sun at midday is 13°. Find the latitude of the point which is on the same meridian as Manchester and where the Sun is directly overhead at midday on that day. How far south of Manchester is this point (measured along the surface of the Earth)?

Question 3

A gym has an exercise machine that is a type of treadmill. The user can choose the gradient to walk or run up. The choice is limited to a small range of slopes, specified in the form '1 in n', where n can be any integer from 5 to 10. It is not clear, however, whether this means that the 'hill' rises 1 metre for every n metres along the slope, or for every n metres travelled horizontally. For both interpretations, make a table of the angle of slope (in degrees) for the various values of n. For each value of n, which interpretation gives the larger value of the slope?

Question 4

A right-angled triangle has sides adjacent to the right angle, which measure 4 and 5 units. What are the angles of the triangle?

Question 5

Look at Figure 1 opposite. A surveyor can find the height, h metres (CD), of a hill and the horizontal distance, x metres, from B to C by measuring the distance, d metres, from A to B and the angles $\alpha = D\widehat{A}B$ and $\beta = D\widehat{B}C$. Calculate the height of the hill when $d = 100$, $\alpha = 10°$ and $\beta = 15°$.

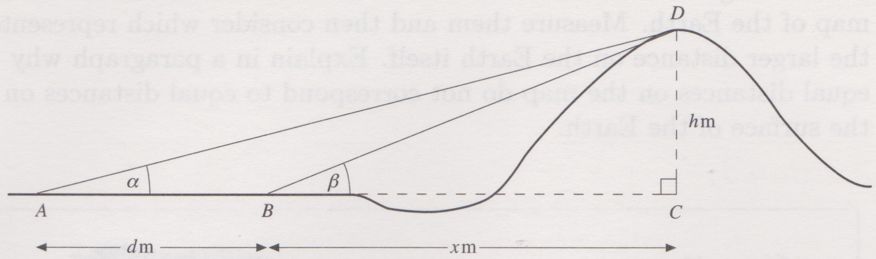


Figure 1

Question 6

While walking along the towpath at the edge of a straight canal, you are struck by a desire to know how wide the canal is. You have a compass, and you know your stride is about 1 metre. You notice a tree growing on the opposite bank a little way ahead. Using your compass, you measure the bearing of the tree from the direction of the towpath: it is 50°. You take five strides back along the towpath and repeat the measurement: the new bearing is 30°. Estimate the width of the canal. (Drawing a diagram will be helpful.)

Your stride might be slightly more (or less) than 1 metre, so how accurate do you think your estimate is?

Question 7

(a) Look at Figure 2. It shows three horizontal lines AB, CD and EF. Which looks to be the shortest, and which the longest? Now measure all three lines. This is a well-known optical illusion. Try it on some other people if you can. Write a paragraph explaining why the illusion occurs.

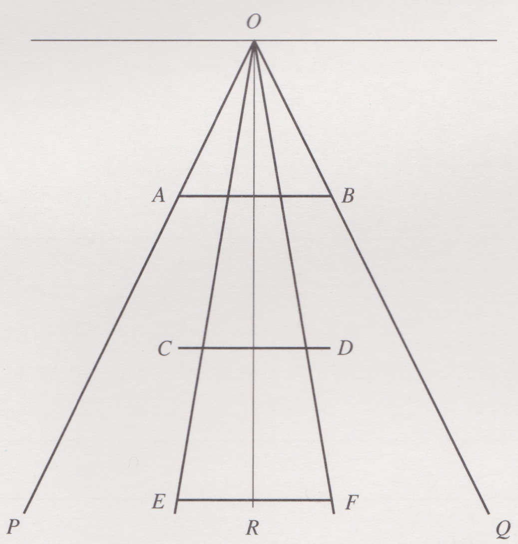


Figure 2

(b) Look at Figure 3. Two horizontal lines AB and CD are drawn on this map of the Earth. Measure them and then consider which represents the larger distance on the Earth itself. Explain in a paragraph why equal distances on the map do not correspond to equal distances on the surface of the Earth.

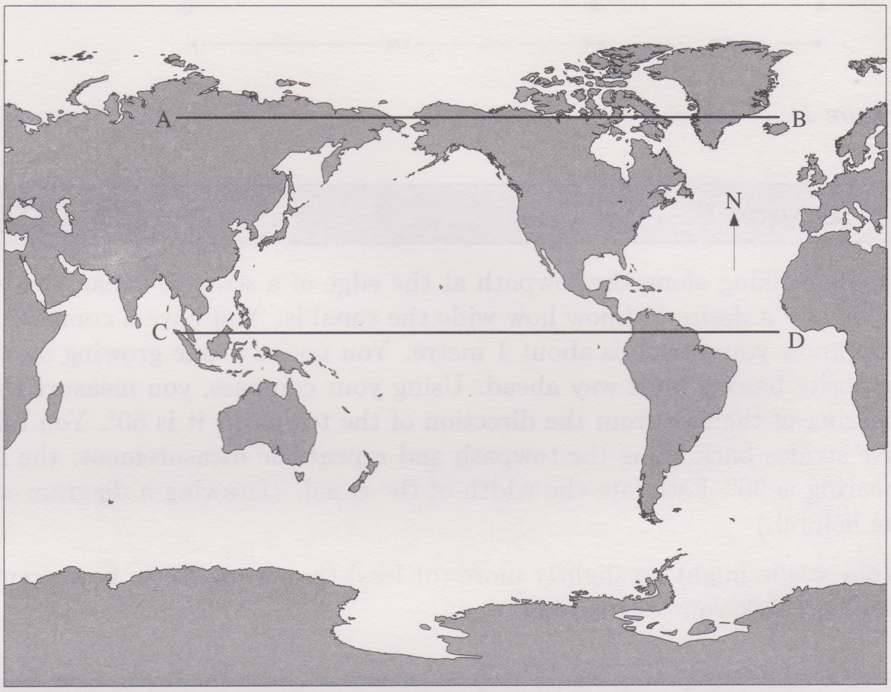


Figure 3

Unit 15 Repeating patterns

Question 1

In the UK, electricity is distributed as alternating current. The electrical voltage at a domestic mains socket can be modelled as a sine wave with a frequency of 50 Hz and an amplitude of about 340 V. What is the formula describing this sine wave?

(The figure of 240 V usually quoted for the mains voltage refers to the 'root-mean-square' value, which is $\frac{1}{\sqrt{2}}$ ($\simeq 0.707$) times the amplitude; this is a form of average value used in electrical engineering.)

Question 2

If two sine waves of the same angular frequency ω are added together, the result is another sine wave of the same frequency but with different amplitude and phase. The result can be expressed in the form $A\sin(\omega t + \phi)$. Use your calculator to plot the sine waves $\sin 10t$ and $\cos 10t$ and their sum. What are the values of A, ω and ϕ when $A\sin(wt + \phi) = \sin 10t + \cos 10t$?

Question 3

(a) $\cos^2 x$ can be expressed in the form $A + B\cos Cx$, where A, B and C are numbers. Display the graph of $\cos^2 x$ on your calculator and hence find the values of A, B and C.

(b) $\sin^2 x$ can be expressed in the form $D + E\cos Fx$, where D, E and F are numbers. Use your calculator to find the values of D, E and F.

(c) Add the results from (a) and (b) (substituting in the values of A, B, C, D, E and F) and simplify as much as possible in order to find a simple expression for $\sin^2 x + \cos^2 x$.

Question 4

Table 1 overleaf gives the times of sunrise and sunset in Liverpool at four-week intervals from early 1996 to early 1997. The times are in Greenwich Mean Time and are expressed in hours and minutes.

(a) Use the data and your calculator to set up a mathematical model for predicting the length of day (from sunrise to sunset) at the beginning of each week in the period covered by Table 1.

(b) How would you modify the model so that it could be used for predicting the length of day (from sunrise to sunset) for any given day in 1996?

Table 1 Sunrise and sunset in Liverpool from early 1996 to early 1997

Week	Date	Sunrise	Sunset	Week	Date	Sunrise	Sunset
0	6 Jan	08.26	16.09	28	20 Jul	04.09	20.26
4	3 Feb	07.54	16.57	32	17 Aug	04.55	19.35
8	2 Mar	06.57	17.52	36	14 Sept	05.44	18.12
12	30 Mar	05.49	18.44	40	12 Oct	06.33	17.22
16	27 Apr	04.45	19.35	44	9 Nov	07.26	16.24
20	25 May	03.57	20.21	48	7 Dec	08.13	15.54
24	22 Jun	03.43	20.44	52	4 Jan	08.26	16.09

Question 5

(a) The trigonometric identity

$$\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right) \tag{1}$$

can be also be written as

$$\sin P + \sin Q = 2\cos X \sin Y, \tag{2}$$

where P and Q are expressions involving X and Y. What are these expressions? Write the identity (2) in terms of X and Y only.

(b) Two pure tones of frequencies P and Q Hz are sounded together and produce beats. The waveform of these beats is described by $2\cos(10t)\sin(80t)$. What are P and Q? Use the values of P and Q to obtain the formula for the beat waveform in the form $y = \sin P + \sin Q$.

Question 6

The Fourier series of a square wave has the form

$$A(\sin\omega_1 t + \frac{1}{3}\sin 3\omega_1 t + \frac{1}{5}\sin 5\omega_1 t + \frac{1}{7}\sin 7\omega_1 t \ + \cdots).$$

(a) The square wave has a period of 1 second. What is the value of the fundamental frequency ω_1?

(b) The value of A can be chosen to fix the amplitude of the square wave. Use your calculator to estimate the value of A that will give a square wave with an amplitude of 1.

Question 7

The Fourier series of a particular periodic waveform has the general form

$$1 + c_2\cos 2\omega_1 t + c_4\cos 4\omega_1 t + \cdots + c_n\cos n\omega_1 t + \cdots,$$

where ω_1 is the fundamental frequency and the values of the coefficients $c_2, c_4, c_6, \ldots$ are given by the expression

$$c_n = \frac{2(^{-}1)^{(1+n/2)}}{(n^2-1)},$$

where n takes the values $2, 4, 6$ and so on.

Work out the values of c_2, c_4 and c_6. Write down the first four terms of the Fourier series for a waveform with a period of 10 milliseconds. Display the waveform on your calculator (over 20 milliseconds). What is the waveform?

Unit 16 Rainbow's end

Question 1

Snell's law of refraction can be written as $\sin r = k \sin i$, where r and i are the angles of refraction and incidence respectively.

(a) Use your calculator to plot a graph of r against i for the following data:

Angle of incidence i/degrees	10	20	30	40	50	60	70	80
Angle of refraction r/degrees	7.5	15	22	29	35	40.5	44.5	47.5

(b) Write Snell's law in the form $r =$ (an expression in k and i).

(c) Enter the function you obtained for r in part (b) into your calculator and store the value 0.5 for k. Plot the predicted graph of r for the incidence angles 10, 20, ... (in degrees), as in part (a). Now try various values of k between 0.5 and 1, and display the predicted graphs of r together with the measured data from the table. Which value of k gives the best fit between Snell's law and the experimental data? Compare your value with Descartes' value of 187/250.

Question 2

In *Paralipomena* (1604) and *Dioptrice* (1611), Kepler published his attempts at finding a trigonometric law of refraction. Some of his formulas, given below, include the difference d between the angle of incidence and the angle of refraction. Replace d by $i - r$ and rearrange each of these formulas into the more conventional form $r =$ (an expression in i and k):

(a) $d = ki$

(b) $d = k/\cos i$

(c) $\tan i = k \tan r$

(d) $\tan i = k \sin d$

(e) $1 - (\tan i/\tan d) = k \tan i$

(f) $1 - (\tan i/\tan d) = k \sin i$.

Question 3

For the rainbow model due to Descartes and Newton, the angle Y can be plotted against the impact parameter X. Each coloured band appears at the angle Y corresponding to the maximum point on its graph, called the Descartes' angle. For any angle Y less than the Descartes' angle, there are *two* corresponding values of the impact parameter.

(a) Plot the graphs of the angle Y against the impact parameter X, where $Y = 4\sin^{-1}(kX) - 2\sin^{-1} X$, for red and violet light, with $k = 81/108$ for red light and $k = 81/109$ for violet light. (You may have already done this in Activity 17 of *Unit 16*.) Find the Descartes' angle and the associated impact parameter for each colour by means of either the zoom or table facilities.

(b) Supernumerary red light (an additional faint arc just inside the primary bow) appears at an angle of about 40.1° (which is just below the angle for violet light in the primary bow). Use the appropriate graph from part (a) to find the two values of the impact parameter X that return red light at this angle.

Units 1–16 Revision

Question 1

Convert each of the following calculations into decimals (to two decimal places). Which **two** produce the same answer?

Options

A $\left(\frac{10}{7}\right)^2$ **B** $\frac{7}{4} + \frac{13}{25}$ **C** $\frac{30}{36.9}$ **D** $\sqrt{(3^2 + 4^2)} \times \frac{20}{7}$

E $\frac{300}{\sqrt{36}}$ **F** $\frac{4}{3} \times \frac{25}{7}$ **G** $\left(\frac{21}{300}\right)^{-1}$ **H** $^{-}2 + 1.11$

Question 2

One light year is the distance travelled by light in one year. This distance is approximately 9.5×10^{12} km. Which **four** options give the distance travelled by light in one second, correct to two significant figures? For the number of days in a year, use 365.

Options

A 3.0×10^8 km **B** 3.0×10^8 m

C 300 million km **D** 300 000 m

E 300 million m **F** 3.0×10^5 km

G 3.0×10^{11} km **H** 300 000 km

Question 3

The eight CMA scores of a student taking a 60-point Open University course are as follows:

80	78	83	94	75	66	86	63

(a) Select the option below that is closest to the mean of the scores in the table.

Options

A 82.4 **B** 77.6 **C** 78.1 **D** 78.7

E 76.7 **F** 79.9 **G** 81.3 **H** 79.2

(b) Select the option below that is closest to the median of the scores in the table.

Options

A 69 **B** 71 **C** 73 **D** 75

E 77 **F** 79 **G** 81 **H** 83

(c) Select the option below that is closest to the standard deviation of the scores in the table.

Options

A 3.1 **B** 4.7 **C** 6.3 **D** 9.5

E 10.2 **F** 12.7 **G** 15.1 **H** 19.8

Question 4

Select the **two** options that are true statements.

Options

A A price ratio of 1.5 is equivalent to a percentage price increase of 15%.

B A price ratio of 0.7 is equivalent to a percentage price decrease of 30%.

C A price ratio of 2.3 is equivalent to a percentage price increase of 23%.

D A percentage price increase of 5.5% is equivalent to a price ratio of 1.55.

E A percentage price decrease of 14% is equivalent to a price ratio of 0.86.

F A percentage price decrease of 20% is equivalent to a price ratio of 1.2.

Question 5

This question is based on the data on computer prices in Table 2 below.

Table 2 Computer price/£s

1440	1570	1470	1750	1430	1495	1555	1505	1395	1465

(a) Select the option below that corresponds to the range of the computer prices in Table 2.

(b) Select the option below that corresponds to the interquartile range of the computer prices in Table 2.

Options

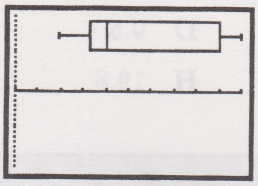
A 115 **B** 105 **C** 355 **D** 125

E 145 **F** 305 **G** 335 **H** None of these

(c) Use your calculator to set up a boxplot for the data in Table 2, with a window for a range of X from 1300 to 1800. Choose the option overleaf that most closely resembles the graph of the boxplot produced by the calculator.

Options

A



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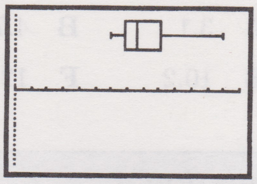

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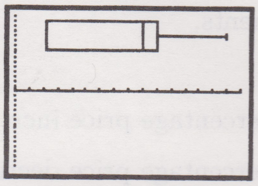

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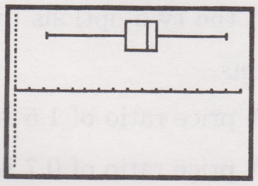

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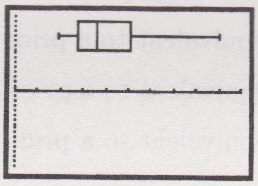

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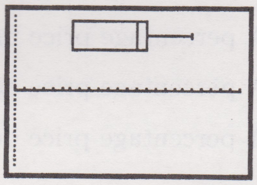

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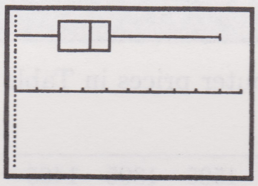
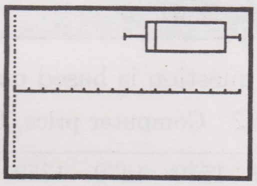


H



Question 6

In September 1997, the Retail Prices Index (RPI) stood at 159.3 (base date January 1987). The Average Earnings Index (AEI) for that month was 139.1 (base year 1990). In September 1996, the RPI was 153.8 and the AEI was 133.3.

(a) Select the option below that corresponds to the percentage increase in average earnings between September 1996 and September 1997 (rounded to one decimal place).

Options

A 0.9	**B** 1.0	**C** 2.4	**D** 3.4
E 4.3	**F** 4.4	**G** 5.8	**H** None of these

(b) In September 1996, the value of an index-linked pension was £88 per week, when adjusted for inflation. Which option below is closest to the value of the weekly pension, when adjusted for inflation, in September 1997?

Options

A £62	**B** £63	**C** £85	**D** £87
E £88	**F** £91	**G** £112	**H** £123

(c) Select the option below that is closest to the real earnings for September 1997 as a percentage of the real earnings one year earlier (rounded to one decimal place).

Options

A 73.1	**B** 99.3	**C** 100.4	**D** 100.7
E 103.4	**F** 103.5	**G** 116.8	**H** 117.2

Question 7

Select the **two** options that correspond to commands which, if used repeatedly, would generate a random sequence of the numbers 1, 2, 3, 4, 5 and 6. Ensure that your calculator is set to FLOAT mode.

Options

A **int(rand)**	B **int(rand *6)**
C **6rand**	D **randInt (0,5)+1**
E **6randInt (0,1)**	F **randInt (0,6)**
G **randInt(1,6)**	H **rand1,6**

Question 8

A map is drawn to a scale of 1 : 10 000. Which option gives the map area that corresponds to a ground area of 80 hectares?

Options

A $0.8\,\text{cm}^2$	**B** $1\,\text{cm}^2$	**C** $1.25\,\text{cm}^2$	**D** $6.4\,\text{cm}^2$
E $8\,\text{cm}^2$	**F** $10\,\text{cm}^2$	**G** $64\,\text{cm}^2$	**H** $80\,\text{cm}^2$

Question 9

Which **two** of the following are true statements?

Options

A In the third quadrant of a graph, the values of x are negative and the values of y are negative.

B The graph of the equation $y = 8x + 2$ passes through the point $(^-2, 14)$.

C The equation $8y = 2x + 8$ represents a straight-line graph with a slope of 2.

D The slope of a straight-line graph can be calculated by dividing the y-coordinate of any point on the line by the x-coordinate of that point.

E Velocity is the mathematical term for speed.

F A negative gradient means that a graph slopes down from right to left.

G Average speed is calculated by multiplying the distance travelled by the journey time.

H The graph of a directly proportional relationship has an intercept of zero.

Question 10

Two pure tones of the same amplitude but with frequencies 16 Hz and 18 Hz respectively are sounded together to produce beats. Each tone is modelled as a sine wave. Which **two** options model the resulting waveform?

Options

A $\sin 16t + \sin 18t$ **B** $\sin 4\pi t + \sin 36\pi t$

C $\sin 2t + \sin 34t$ **D** $\sin 32\pi t + \sin 36\pi t$

E $2\cos(16t)\sin(18t)$ **F** $2\cos(4\pi t)\sin(32\pi t)$

G $2\cos(2\pi t)\sin(34\pi t)$ **H** $2\cos(32\pi t)\sin(36\pi t)$

Question 11

Select the option that is equivalent to 225° in radians.

Options

A $\frac{2}{3}\pi$ **B** $\frac{3}{4}\pi$ **C** $\frac{3}{8}\pi$ **D** $\frac{5}{2}\pi$

E $\frac{5}{4}\pi$ **F** $\frac{5}{8}\pi$ **G** $\frac{7}{2}\pi$ **H** None of these

Question 12

The formula

$$y = a - 2bx^2$$

is rearranged to make x the subject. Select the option that gives the correct rearranged formula.

Options

A $x = \left(\dfrac{y-a}{2}\right) - \sqrt{b}$ **B** $x = (y-a)/\sqrt{2b}$

C $x = \dfrac{b}{2}\sqrt{y-a}$ **D** $x = \dfrac{\sqrt{b(y-a)}}{2}$

E $x = \sqrt{2(y-a)/2b}$ **F** $\sqrt{\dfrac{a-y}{2b}}$

G $\sqrt{\dfrac{y-a}{2b}}$ **H** None of these

Question 13

Use your calculator to solve the following equation:

$$5x^3 + 12x = 11.$$

Select the option that is closest to the solution.

Options

A 0.74 **B** 0.75 **C** 0.76 **D** 0.77

E 0.78 **F** 0.79 **G** 0.80 **H** 0.81

Question 14

Which **three** of the following are true statements?

Options

A The solution of the equation $10^x = 100$ is 10.

B An exponentially growing population with a doubling time of 20 years will increase by 50% in 10 years.

C The expression $\sqrt[3]{x^{12}}$ is another way of writing x^4.

D Each year, 1% of the atoms in a block of a radioactive substance decay. This means that the number of atoms in the block is decreasing exponentially.

E A monthly interest rate of 2% is equivalent to an APR of 24%.

F The surface area of a sphere is directly proportional to the square of its volume.

G The radius of a sphere is directly proportional to the cube of its volume.

H The radius of a sphere is directly proportional to the cube root of its volume.

Question 15

Table 3 below shows how the duration of a £50 000 mortgage is related to the required monthly payments, provided the interest rate remains unchanged during the course of the mortgage.

Table 3 Mortgage repayments

Duration of mortgage in years (X)	25	21.5	18	15	12.5
Monthly payment in £s (Y)	361.12	386.12	411.12	461.12	511.12

Input the data into your calculator and use the regression facilities to find the equation for the curve of best fit, with the coefficients rounded to three significant figures.

Select the **four** options that are true statements.

Options

A Power regression gives the best fit.

B Linear regression gives the best fit.

C Exponential regression gives the best fit.

D The power regression equation is $Y = 1790X^{-0.501}$.

E The power regression equation is $Y = 1791X^{-0.5008}$.

F The exponential regression equation is $Y = 699 \times (0.973)^X$.

G Paying off the mortgage at £450 per month will take about 16 years.

H Paying off the mortgage at £500 per month will take 10 years.

Question 16

The function $y = 25 + 45\exp(^-0.14t)$ can be used to model the way that the temperature y°C of water in a domestic hot-water tank varies with the time t hours since the immersion heater was switched off. Select the **two** options that are correct interpretations of this model.

Options

A The water is cooling towards a room temperature of 20°C.

B The water is cooling towards a room temperature of 25°C.

C The water is cooling towards a room temperature of 45°C.

D The water is cooling towards a room temperature of 70°C.

E The initial temperature of the water was 20°C.

F The initial temperature of the water was 25°C.

G The initial temperature of the water was 45°C.

H The initial temperature of the water was 70°C.

Question 17

The lengths of the sides of a triangle are 12, 8 and 16 units. Each of the options below gives the lengths of the sides of other triangles. Select the **two** options that correspond to triangles which are similar to the first triangle.

Options

A $5\frac{1}{2},\ 2\frac{1}{2},\ 3\frac{3}{4}$ **B** 4, 8, 6 **C** $2\frac{1}{2},\ 3\frac{1}{2},\ 4\frac{1}{2}$

D $5\frac{1}{3},\ 6\frac{2}{3},\ 3\frac{1}{3}$ **E** $\frac{1}{4},\ \frac{3}{8},\ \frac{1}{2}$ **F** 50, 75, 125

G 10, 5, 5

Question 18

Compass bearings of a cairn are taken from two points 1 km apart along a straight north–south path. After adjustment for magnetic variation, the bearings are 15° and 75°. Select the **two** options that are nearest to the distances (in metres) from the two points to the cairn.

Options

A 77 **B** 299 **C** 325 **D** 648

E 966 **F** 1092 **G** 1115 **H** 1301

Question 19

In Exercise 14.7 of the *Calculator Book*, the latitudes and longitudes of six cities are given. Use these data to determine which option below gives the great circle distance, to the nearest 1000 km, from Bangkok to New York.

Options

A 10 000 **B** 11 000 **C** 12 000 **D** 13 000

E 14 000 **F** 15 000 **G** 16 000 **H** 17 000

Question 20

The arc of the great circle joining Milton Keynes to Bangkok subtends an angle of about 100° at the centre of the Earth. Select the option that corresponds most closely to the distance between Milton Keynes and Bangkok when measured over the surface of a terrestrial globe of radius 1 m.

A 0.5 m **B** 0.9 m **C** 1.1 m **D** 1.4 m

E 1.7 m **F** 1.9 m **G** 2.2 m **H** 2.7 m

Question 21

A sine wave has an amplitude of 20, a period of 0.5 s and a relative phase shift of 45°. Which option represents this sine wave?

Options

A $20\sin(2\pi t + \pi/4)$ **B** $20\sin(4\pi t + \pi/4)$ **C** $0.5\sin(20t + \pi/4)$

D $0.5\sin(0.5t + 45)$ **E** $0.5\sin(20\pi t + 45)$ **F** $20\sin(2t + \pi/4)$

G $20\sin(4t + \pi/4)$ **H** None of these

Question 22

Table 4 gives the times of sunrise in Glasgow at four-week intervals. The times are in Greenwich Mean Time and are expressed in hours and minutes.

Table 4 Sunrise in Glasgow

Week	Sunrise	Week	Sunrise
0	08.46	28	04.01
4	08.09	32	04.53
8	07.05	36	05.47
12	05.52	40	06.42
16	04.42	44	07.41
20	03.48	48	08.33
24	03.31	52	08.46

Select the option that provides the best fit to the sunrise data, where x represents the week number (round coefficients to two significant figures).

Options

A $6.2\sin(0.12x + 1.9) + 2.6$

B $6.2\sin(1.9x + 0.12) + 2.6$

C $2.6\sin(0.12x + 1.9) + 6.2$

D $2.6\sin(1.9x + 0.12) + 6.2$

E $1.9\sin(0.12x + 6.2) + 2.6$

F $0.12\sin(1.9x + 2.6) + 6.2$

Question 23

Select the **two** options that are true for all values of x.

Options

A $\sin x = \sin(x + 5\pi)$

B $\cos(x + \pi/2) = {}^{-}\cos x$

C $\sin(x) = {}^{-}\sin({}^{-}x)$

D $\cos x = {}^{-}\sin(x + \pi/2)$

E $\sin(x + \pi) = \cos(x + 3\pi/2)$

F $\cos(x + 4\pi) = \cos(x + 2\pi)$

G $\sin({}^{-}x) = \sin x$

H $\cos({}^{-}x) = {}^{-}\cos x$

Question 24

Which **four** of the following statements about the mathematical model of the rainbow are true?

Options

A The angle of the primary bow is about 51°.

B The angle of the primary bow is about 42°.

C The secondary bow is formed by the reflection and refraction of sunlight.

D The primary bow is formed only by the refraction of sunlight.

E On entering a raindrop, red light is refracted more than violet light.

F On entering a raindrop, violet light is refracted more than red light.

G The angle between the primary and secondary bows is about 9°.

H The angle between the primary and secondary bows is about 42°.

Question 25

Which **three** of the following are *incorrect* programming commands for the TI-83 calculator?

Options

A **:DispGraph**

B **:For(S,10,0,2)**

C **:If V<.175**

D **:Lbl 0**

E **:Disp L3,L2,A**

F **:Fix 0.5**

G **:Plot1(Boxplot, L1)**

H **:Input X,Y**

Question 26

An MU120 student produced the following program to calculate cooking times for joints of meat.

Program : COOKING
:Input "K?", K
:Input "T?", T
:Disp (K × 2.2) × T

Which **four** of the following statements are true?

Options

A The program calculates the cooking time for a joint whose weight is given in kilograms when the recipe book gives the time per pound.

B The program calculates the cooking time for a joint whose weight is given in pounds when the recipe book gives the time per kilogram.

C The cooking time must be entered in minutes.

D The cooking time must be entered in hours.

E The brackets in the last line of the program are necessary.

F The brackets in the last line of the program are unnecessary.

G The program is based upon the assumption that weight in kilograms is 2.2 times weight in pounds.

H The program is based upon the assumption that cooking time is proportional to weight.

Solutions

Unit 14

1

(a) The relative sizes of parts of the body change as people age; in particular, a toddler's head is larger in relation to the rest of his or her body than an adult's. The bodies of an adult and a toddler are therefore not similar.

(b) Chessboards, being square, are all similar to each other. Moreover, since each chessboard is subdivided into the same number of smaller squares, chessboards are also similarly patterned.

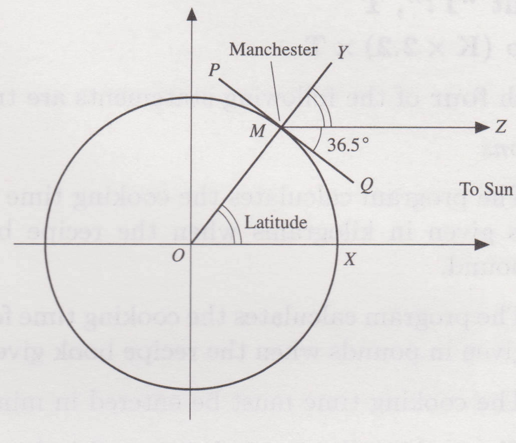
(c) The hypotenuse (in cm) of the first triangle is $\sqrt{4^2 + 5^2} = \sqrt{41}$, while that of the second is $\sqrt{12^2 + 15^2} = \sqrt{369} = 3\sqrt{41}$. So the sides of the larger triangle are all three times the length of those of the smaller one, hence the triangles are similar.

Alternatively, note that corresponding sides of the two triangles are in proportion and that the included angle is the same (a right angle in each case).

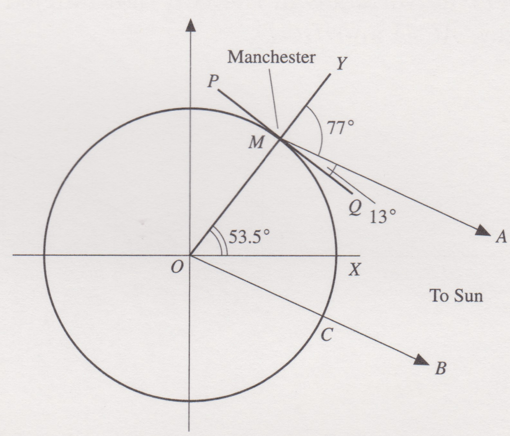
(d) If the Earth and Moon were both perfect spheres, then they would be similar. However, the Earth is not perfectly spherical, but is flattened at its poles. For the two bodies to be similar, the Moon would have to have this same feature to the same degree proportionally, which seems highly unlikely. The Earth and the Moon are therefore not exactly similar. (Further investigation of reference books will confirm that this answer is correct.)

2

(a) At midday on 21 March, a line MZ drawn from Manchester towards the Sun will be parallel to a line OX, such that O is at the centre of the Earth and X is a point on the Equator, lying on the same meridian as Manchester. The Sun is overhead at X. The line OMY represents the extended radius of the Earth through Manchester, and will be vertical for people in Manchester, while the line PMQ will be horizontal. The angle $Y\hat{M}Z$ at Manchester is the same as the angle $M\hat{O}X$, which is the latitude of Manchester. Since the angle of elevation of the Sun (its angle above the tangent to the Earth's surface) is 36.5°, the angle between the vertical OMY and the Sun's direction is $(90 - 36.5)° = 53.5°$. So the latitude of Manchester is 53.5°N.

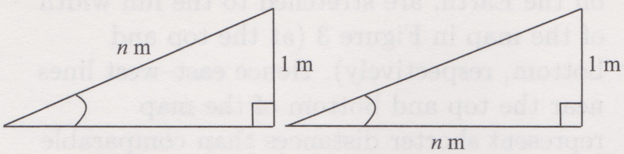


(b) At midday on 21 December, the line MA drawn from Manchester towards the Sun makes an angle of $(90 - 13)° = 77°$ with the vertical. Now, MA is parallel to OCB, the line from the centre of the Earth through C, the point on Manchester's meridian at which the Sun is overhead at midday. The latitude of C(that is, angle $X\hat{O}C$) is therefore $M\hat{O}C - M\hat{O}X = 77° - 53.5° = 23.5°$S (since $M\hat{O}C = Y\hat{M}A = 77°$). Thus C lies on the tropic of Capricorn, which is the line of latitude approximately 23.5°S of the Equator.



The distance of C from Manchester is the distance corresponding to an angle of 77° ($Y\hat{O}C$) on the meridian, that is, $77/360 \times 2\pi \times 6368 \simeq 8558$ km (with the Earth's radius being taken to be 6368 km).

3



To find the angle corresponding to 'n metres along the slope', you must calculate $\sin^{-1}(1/n)$; while for 'n metres horizontally', $\tan^{-1}(1/n)$ is required. You could use your calculator to find these values by entering the functions $\mathrm{Y1} = \sin^{-1}(1/\mathrm{X})$ and $\mathrm{Y2} = \tan^{-1}(1/\mathrm{X})$, and then using the table facility. The results (in degrees, to 2 d.p.) are given in the table below.

n	$\sin^{-1}(1/n)$	$\tan^{-1}(1/n)$
5	11.54	11.31
6	9.59	9.46
7	8.21	8.13
8	7.18	7.13
9	6.38	6.34
10	5.74	5.71

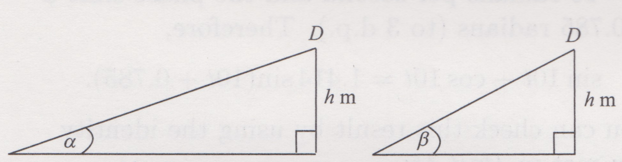
It is clear that $\sin^{-1}(1/n)$ always has the larger value, so the gradient is always greater when n is measured *along* the slope.

4

One angle is 90° (the right angle). The angle opposite the side of length 4 is given by $\tan^{-1}(4/5) \simeq 38.66°$. The angle opposite the side of length 5 is therefore (approximately) $(90 - 38.66)° = 51.34°$.

5

Trigonometry can be used to find relationships between the variables in the two right-angled triangles ACD and BCD.



Thus

$$\tan\alpha = h/(d+x)$$

and

$$\tan\beta = \frac{h}{x}$$

Algebra can be used to find h and x if d, α and β are known. There are several ways of doing the calculations. Here is one way.

As $d = 100$ and $\alpha = 10°$,

$$\tan\ 10° = \frac{h}{100+x},$$

so

$$(100 + x)\tan\ 10° = h$$

or

$$h = 100\tan\ 10° + x\tan\ 10°. \qquad (1)$$

As $\beta = 15°$,

$$\tan\ 15° = \frac{h}{x},$$

so

$$h = x\tan\ 15°. \qquad (2)$$

Now (1) and (2) are both expressions for h and so must be equal. Thus

$$100\ \tan\ 10° + x\tan\ 10° = x\tan\ 15°$$

$$100\ \tan\ 10° = x(\tan\ 15° - \tan\ 10°)$$

$$x = \frac{100\ \tan\ 10°}{(\tan\ 15° - \tan\ 10°)} = 192.45\ldots.$$

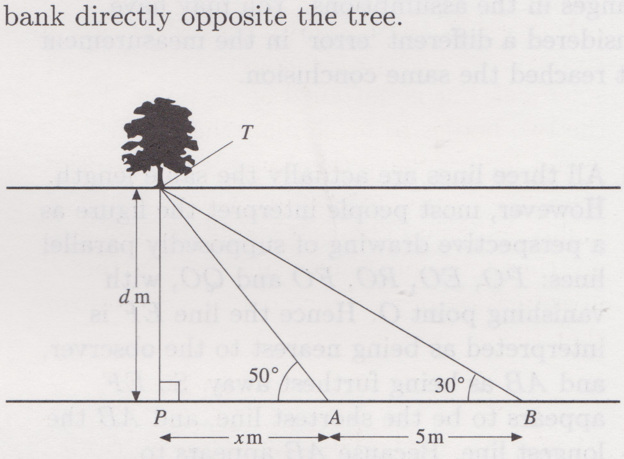
Substituting for x in (2) gives

$$h = 51.5668\ldots.$$

So the hill is about 52 m high (and B is about 190 m from C).

6

Call the point where the tree stands T. Let the point where you first measured the bearing be A, the point where you measured the second bearing be B, and let P be the point on the bank directly opposite the tree.



Let the width of the canal be d metres, and the distance AP be x metres. Then in triangle APT,

$$\tan 50° = \frac{d}{x},$$

so

$$x = d/\tan\ 50°, \qquad (1)$$

and in triangle BPT,

$$\tan 30^\circ = \frac{d}{x+5},$$

so

$$x + 5 = d/\tan\ 30^\circ. \qquad (2)$$

Substituting x from (1) into (2) gives

$$d/\tan 50^\circ + 5 = d/\tan 30^\circ,$$

so

$$\begin{aligned} 5 &= (d/\tan\ 30^\circ) - (d/\tan\ 50^\circ) \\ &= d(1/\tan\ 30^\circ - 1/\tan\ 50^\circ), \end{aligned}$$

hence

$$\begin{aligned} d &= 5/(1/\tan\ 30^\circ - 1/\tan\ 50^\circ) \\ &\simeq 5.6. \end{aligned}$$

Therefore the canal is about 5.6 m wide.

If your stride was such that the measurement of AB was 0.1 m more (or less), then AB would be 5.1 m (or 4.9 m) instead of 5 m. So

$$\begin{aligned} d &= 5.1/(1/\tan 30^\circ - 1/\tan 50^\circ) \\ &= 5.7. \end{aligned}$$

Repeating the calculation with $AB = 4.9$ gives

$$d = 5.5.$$

Thus your answer is reasonably accurate for a rough estimate, and is fairly stable under slight changes in the assumptions. You may have considered a different 'error' in the measurement but reached the same conclusion.

7

(a) All three lines are actually the same length. However, most people interpret the figure as a perspective drawing of supposedly parallel lines: PO, EO, RO, FO and QO, with vanishing point O. Hence the line EF is interpreted as being nearest to the observer, and AB as being furthest away. So EF appears to be the shortest line, and AB the longest line. Because AB appears to represent a line of the same length as PQ, we are tempted to see it as about three times the length of EF.

(b) The two lines are of equal length, but CD represents an east–west distance near the Equator, whereas AB represents an east–west distance near the Arctic. So CD represents a longer distance than AB, as explained below.

Because the Earth is a sphere, it cannot be represented accurately on a flat page. There has to be some distortion. Thus, the North and South Poles, which in reality are points on the Earth, are stretched to the full width of the map in Figure 3 (at the top and bottom, respectively). Hence east–west lines near the top and bottom of the map represent shorter distances than comparable lines near the Equator.

A similar effect holds for areas: Australia is not smaller than Greenland as it appears on this map, but over three times larger!

Unit 15

1

The general formula for a sine wave is $A\sin(\omega t + \phi)$, where A is the amplitude, ω is the angular frequency and ϕ is the relative phase shift. In this case, $A = 340$ V and $\omega = 2\pi f$, where the frequency $f = 50$ Hz, so $\omega = 100\pi$ (or about 314.2 radians per second). Take ϕ to be zero (since the time origin has not been specified). So the voltage V (in volts) at the mains socket is modelled by the formula

$$V = 340\sin(100\pi t).$$

2

If you display the sum $\sin 10t + \cos 10t$ on your calculator, you should find that the result is another sine wave with the same angular frequency (10 radians per second) and an amplitude of about 1.414. If you plot the sine curve $1.414\sin 10t$, however, you should find that it lags behind the curve of the sum by about 0.125 of a period. This phase lag corresponds to a phase shift of $0.125 \times 2\pi \simeq 0.785$ (radians). So $A = 1.414$, $\omega = 10$ radians per second and the phase shift ϕ is 0.785 radians (to 3 d.p.). Therefore,

$$\sin 10t + \cos 10t = 1.414\sin(10t + 0.785).$$

You can check this result by using the identity you met in *Unit 15*:

$$\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right).$$

Now $\cos 10t$ is identical to $\sin(10t + \pi/2)$. So $\sin 10t + \cos 10t$ becomes $\sin 10t + \sin(10t + \pi/2)$, hence $a = 10t$ and $b = 10t + \pi/2$. Then the identity becomes

$$\begin{aligned} &\sin 10t + \sin(10t + \pi/2) \\ &= 2\cos\left(\frac{10t - 10t - \pi/2}{2}\right)\sin\left(\frac{10t + 10t + \pi/2}{2}\right) \\ &= 2\cos\left(-\frac{\pi}{4}\right)\sin\left(10t + \frac{\pi}{4}\right). \end{aligned}$$

But

$$2\cos\left(-\frac{\pi}{4}\right) = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2},$$

so the identity can be written as

$$\sin 10t + \cos 10t = \sqrt{2}\sin\left(10t + \frac{\pi}{4}\right)$$
$$\simeq 1.414\sin(10t + 0.785).$$

3

(a) You should find that the graph of $\cos^2 x$ is itself a cosine curve, which varies above and below a mean value of 0.5, with an amplitude of 0.5. It completes two cycles for each complete cycle of $\cos x$. So $A = 0.5$, $B = 0.5$ and $C = 2$, giving the following trigonometric identity:

$$\cos^2 x = 0.5 + 0.5\cos 2x.$$

You can check this by plotting the function $\text{Y1} = \cos^2 \text{X}$ and $\text{Y2} = 0.5 + 0.5\cos 2\text{X}$ on your calculator.

(b) You should find that the graph of $\sin^2 x$ also varies above and below a mean value of 0.5, with an amplitude of 0.5, and that it completes two cycles for each complete cycle of $\sin x$. This time, however, the curve is the negative of a cosine curve. So $D = 0.5$, $E = {}^{-}0.5$ and $F = 2$, giving the following identity:

$$\sin^2 x = 0.5 - 0.5\cos 2x.$$

(c) Adding the two curves together gives

$$\sin^2 x + \cos^2 x = 0.5 - 0.5\cos 2x + 0.5 + 0.5\cos 2x$$
$$= 1.$$

This is an important general result, as it is true for *all* values of x.

4

(a) The data given in the Table 1 cover a period of 52 weeks.

The day length is found by subtracting the time of sunrise from the time of sunset. This can be done by entering the data into lists on the calculator (remembering to convert from hours and minutes to a decimal) and using the list arithmetic operations. Then use sine regression (Sin Reg) on your calculator to give the model for the day length y (hours) in terms of the week x:

$$y = 4.60\sin(0.12x - 1.25) + 12.20$$

(coefficients to 2 d.p.).

(b) It is necessary to change the time scale from weeks to days. There are seven days in the week, and week 0 started on the the sixth day of the year. So the x-week list needs to be replaced by a days list where $d = 6 + 7x$. You can do this either by list arithmetic and repeating the sine regression, or you can replace x in the model using $x = (d - 6)/7$ (from $d = 6 + 7x$).

The new model for the day length y (hours) in terms of the day of the year d is

$$y = 4.60\sin(0.017d - 1.35) + 12.20,$$

where d ranges from 1 to 366 (1996 was a leap year).

5

(a) Comparing the two identities gives $P = a$, $Q = b$, $X = (a - b)/2$ and $Y = (a + b)/2$. Substituting for a and b in the expressions for X and Y yields

$$X = \frac{P - Q}{2} \quad \text{and} \quad Y = \frac{P + Q}{2}.$$

Then, adding gives

$$X + Y = \frac{P - Q}{2} + \frac{P + Q}{2}$$
$$= \frac{P}{2} - \frac{Q}{2} + \frac{P}{2} + \frac{Q}{2}$$
$$= P.$$

Similarly, subtracting gives

$$X - Y = \frac{P - Q}{2} - \frac{P + Q}{2}$$
$$= \frac{P}{2} - \frac{Q}{2} - \frac{P}{2} - \frac{Q}{2}$$
$$= {}^{-}Q$$

So $P = X + Y$ and $Q = Y - X$, and the identity (2) can be written as

$$\sin(Y + X) + \sin(Y - X) = 2\cos X \sin Y.$$

(b) By comparison with the identity obtained in part (a), it follows that, for the given waveform $2\cos(10t)\sin(80t)$, $X = 10t$ and $Y = 80t$. Now $P = X + Y = 90t$ and $Q = Y - X = 70t$. Therefore the formula for the beat waveform can be written as

$$y = \sin 90t + \sin 70t.$$

6

(a) Angular frequency ω is related to the period T by the formula $\omega = 2\pi/T$. Since $T = 1$ second for the square wave, the fundamental frequency $\omega_1 = 2\pi/1 = 6.263$ radians per second.

(b) If you set $A = 1$ and use your calculator program FOURIER to plot the square wave from the Fourier series, you should find that the amplitude (that is, the height of the flat top of the waveform) approaches a value of about 0.785. To obtain an amplitude of 1, therefore, the Fourier series must be scaled by the factor $A = 1/0.785 = 1.274$.

[It turns out that the value of A can be calculated from the theory of Fourier series. It is directly proportional to the amplitude, and, for an amplitude X, the value of A is $4X/\pi$. Thus, for a square wave of amplitude 1, the value of A is $4/\pi$, or 1.273 (to 3 d.p.).]

7

For $n = 2$, the exponent $(1 + n/2) = 2$, so

$$c_2 = \frac{2(^-1)^2}{(4-1)} = \frac{2}{3}.$$

For $n = 4$, the exponent $(1 + n/2) = 3$, so

$$c_4 = \frac{2(^-1)^3}{(16-1)} = \frac{^-2}{15}.$$

For $n = 6$, the exponent is 4, so

$$c_6 = \frac{2(^-1)^4}{(36-1)} = \frac{2}{35}.$$

Angular frequency $\omega = 2\pi/T$, where T is the period. Hence, for $T = 10$ milliseconds (10×10^{-3} s), the fundamental frequency $\omega_1 = 2\pi/(10 \times 10^{-3}) = 200\pi$ radians per second. Therefore the Fourier series is

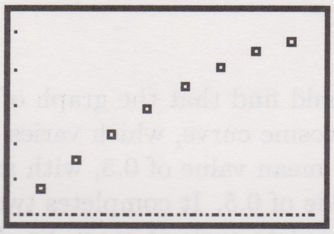
$$1 + \frac{2}{3}\cos 400\pi t$$
$$- \frac{2}{15}\cos 800\pi t + \frac{2}{35}\cos 1200\pi t - \cdots.$$

The calculator display shows that the waveform is a cosine curve with the negative peaks replaced by positive peaks (sometimes called a 'rectified' cosine).

Unit 16

1

(a) The answer is provided by your calculator screen and should look like the screen dump below.



(b) Snell's law can be written in the form:

$$r = \sin^{-1}(k \sin i).$$

Note: this is *not* the same as $\sin^{-1} k \cdot \sin i$.

(c) You should find that a value of $k \simeq 0.75$ gives the best fit. Descartes' value is 0.748.

2

Substituting for d, as appropriate, and rearranging gives the following.

(a) $i - r = ki$,

so

$r = i - ki$ or $i(1-k)$

(b) $i - r = k/\cos i$,

so

$r = i - k/\cos i$.

(c) $\tan r = (\tan i)/k$,

so

$r = \tan^{-1}[(\tan i)/k]$.

(d) $\tan i = k \sin(i - r)$

$\sin(i - r) = (\tan i)/k$

$i - r = \sin^{-1}[(\tan i)/k]$,

so

$r = i - \sin^{-1}[(\tan i)/k]$.

(e) $1 - [\tan i/\tan(i-r)] = k \tan i$

$1 - k\tan i = \tan i/\tan(i-r)$

$$\tan(i-r) = \frac{\tan i}{1 - k\tan i}$$

$$i - r = \tan^{-1}\left(\frac{\tan i}{1 - k\tan i}\right)$$

so

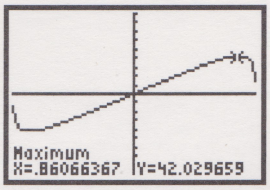
$$r = i - \tan^{-1}\left(\frac{\tan i}{1 - k\tan i}\right).$$

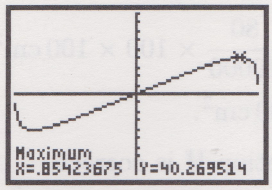
(f) As in (e), but replace $k \tan i$ by $k \sin i$, so

$$r = i - \tan^{-1}\left(\frac{\tan i}{1 - k\sin i}\right)$$

3

(a)





For red light, the Descartes' angle is $42.03°$ (0.73356 radians) and the associated impact parameter X is 0.861. For violet light, the Descartes' angle is $40.27°$ (0.70284 radians) and the associated impact parameter X is 0.854.

(b) Red light is returned at $40.1°$ by impact parameters $X = 0.761$ and 0.929.

Units 1–16

1

A	2.04 (to 2 d.p.)	**B**	2.27 (to 2 d.p.)
C	0.81 (to 2 d.p.)	**D**	14.29 (to 2 d.p.)
E	50	**F**	4.76 (to 2 d.p.)
G	14.29 (to 2 d.p.)	**H**	$^{-}0.89$

Options D and G are the same.

Reference: *Calculator Book*, Chapter 1.2, 1.4 and 1.7.

2

Number of seconds in a year

$= 365 \times 24 \times 60 \times 60.$

Therefore, distance travelled by light in one second

$= 9.5 \times 10^{12} \div (365 \times 24 \times 60 \times 60)$ km

$= 301243.0238$ km

$= 300\,000$ km (to 2 s.f.) (option H)

$= 3 \times 10^5$ km (option F)

$= 3 \times 10^5 \times 10^3$ m $= 3 \times 10^8$ m (option B)

$= 300 \times 10^6$ m $= 300$ million m (option E).

Options B, E, F and H are correct.

Reference: *Calculator Book*, Chapter 1.6.

3

Enter the data in a list in your calculator and use 1-Var Stats (on STAT Calc).

(a) Mean, $\overline{x} = 78.125$.
Option C is correct.

(b) Median $= 79$.
Option F is correct.

(c) Standard deviation, $\sigma_x = 9.53$.
Option D is correct.

Reference: *Calculator Book*, Chapters 2.1 and 4.3.

4

A Not true: a price ratio of 1.5 is equivalent to a price increase of 50%.

B True

C Not true: a price ratio of 2.3 is equivalent to a price increase of 130%.

D Not true: a price increase of 5.5% is equivalent to a price ratio of 1.055.

E True

F Not true: a price *decrease* of 20% is equivalent to a price ratio of 0.8.

Reference: *Unit 2*, Section 4.

5

(a) Inputting the data into a list in the calculator and ordering them (or plotting a boxplot) gives the highest price (in £s) as 1750 and the lowest as 1395. So the range is $1750 - 1395 = 355$.

Option C is correct.

(b) The quartiles are 1440 and 1555. So the interquartile range is $1555 - 1440 = 115$.

Option A is correct.

(c) The boxplot (with the specified window) looks like that in option E.

Reference: *Calculator Book*, Chapter 3.

6

(a) The ratio of the average earnings is

$$\frac{\text{AEI (Sept 1997)}}{\text{AEI (Sept 1996)}} = \frac{139.1}{133.3} = 1.04351.$$

As a percentage, this is 104.4% (to 1 d.p.) or an increase of 4.4%.

Option F is correct.

(b) The price ratio that is relevant here is

$$\frac{\text{RPI (Sept 1997)}}{\text{RPI (Sept 1996)}} = \frac{159.3}{153.8} (= 1.03576\ldots).$$

The index-linked pension will be increased in this proportion:

$$£88 \times \frac{159.3}{153.8} = £91.1\ldots$$
$$= £91 \text{ (to the nearest £)}.$$

Option F is correct.

(c) Real earnings take into account the RPI. Thus

$$\frac{\text{real earnings (Sept 1997)}}{\text{real earnings (Sept 1996)}}$$
$$= \frac{\text{AEI (Sept 1997)}}{\text{AEI (Sept 1996)}} \times \frac{\text{RPI (Sept 1996)}}{\text{RPI (Sept 1997)}}$$
$$= \frac{139.1}{133.3} \times \frac{153.8}{159.3} = 1.00748\ldots.$$

As a percentage, this is 100.7% (to 1 d.p.).

Option D is correct.

Reference: *Unit 3*, Section 7.

7

A Incorrect: always produces 0.

B Incorrect: produces a random sequence of the integers 0 to 5.

C Incorrect: does not produce integers.

D Correct

E Incorrect: produces a random sequence of just 0 and 6.

F Incorrect: produces a random sequence of 0 to 6.

G Correct

H Incorrect: gives a syntax error.

Reference: *Calculator Book*, Chapter 4.2.

8

One hectare is 100 m by 100 m.

On the map, 1 cm represents 10 000 cm or 100 m. So 1 cm^2 represents 100 m by 100 m, or 1 hectare. Therefore, 80 hectares are represented by 80 cm^2.

Alternatively,

80 hectares $= 80 \times 100 \times 100\,\text{m}^2$.

On the map, this is represented by

$$\frac{80 \times 100 \times 100\,\text{m}^2}{(10000)^2} = \frac{80}{10000}\,\text{m}^2$$
$$= \frac{80}{10000} \times 100 \times 100\,\text{cm}^2$$
$$= 80\,\text{cm}^2.$$

Option H is correct.

Reference: *Unit 6*, Section 4.4.

9

A True

B Not true: if $x = {}^-2$, then $y = 8x + 2 = {}^-16 + 2 = {}^-14$ (not 14).

C Not true: $8y = 2x + 8$ does represent a straight-line graph, but rewriting the equation as $y = \frac{1}{4}x + 1$ shows that its slope is $\frac{1}{4}$.

D Not true: this holds only if the graph passes through the origin.

E Not true: velocity has a direction, whereas speed does not.

F Not true: it slopes down from left to right.

G Not true: *divide* the distance travelled by the average journey time rather than multiply.

H True: the graph passes through (0,0) and hence the intercept is zero.

Reference: *Unit 7*, Section 1.3, and *Unit 10*, Section 1.

10

A pure tone of frequency 16 Hz can be modelled by $\sin(16 \times 2\pi t) = \sin 32\pi t$, and similarly a pure tone of frequency 18 Hz by $\sin 36\pi t$. So the resulting waveform is given by $\sin 32\pi t + \sin 36\pi t$.

Option D is correct.

Plotting the functions on the same graph shows that this is the same function as $2\cos(2\pi t)\sin(34\pi t)$.

Option G is correct.

Reference: *Calculator Book*, Chapter 9.2.

11

As $180° = \pi$ radians, $1° = \dfrac{\pi}{180}$ radians.

So $225° = \dfrac{225\pi}{180} = \dfrac{25\pi}{20} = \dfrac{5\pi}{4}$ radians.

Option E is correct.

Reference: *Unit 9*, Section 1, and *Unit 15*, Section 1.

12

$y = a - 2bx^2$,

so

$2bx^2 = a - y$.

Hence

$x^2 = \dfrac{a-y}{2b}$,

or

$x = \sqrt{\dfrac{a-y}{2b}}$.

Option F is correct.

Reference: *Unit 8*, Section 4.

13

Enter $Y = 5x^3 + 12x$ and find where $Y = 11$ is on the graph or table. Alternatively, enter $Y = 5x^3 + 12x - 11$ and find where $Y = 0$ is on the graph or table.

In both cases you should find that the solution occurs where $x = 0.7446$ (to 4 s.f.).

Option A is correct.

Reference: *Calculator Book*, Chapter 8.5.

14

A Not true: the solution is $x = 2$ since $10^2 = 100$.

B Not true: if a population increased by 50% every 10 years, it would be $(1.5)^2 = 2.25$ times larger after 20 years and so would have *more* than doubled.

C True: $x^{12} = x^4 \times x^4 \times x^4$, so $\sqrt[3]{x^{12}} = x^4$.

D True

E Not true: a monthly interest rate of 2% corresponds to 1.02 per month, and $(1.02)^{12} = 1.268$ per year (12 months) or an APR of 26.8%.

F Not true: the surface area of a sphere is directly proportional to the square of the radius *not* the square of the volume.

G Not true: the volume is directly proportional to the cube of the radius *not* vice versa.

H True

Reference: *Unit 12*, Sections 4 and 5, and *Unit 13*, Section 1.

15

Put the duration (X) of the mortgage in list L1 and the monthly payments (Y) in list L2, then performing power regression gives $Y = aX^b$,

where $a = 1790.535278$

and $b = {}^{-}0.5007843558$,

with the regression coefficient $r = {}^{-}0.9946748577$. Rounding a and b to 3 s.f. gives $Y = 1790X^{-0.501}$. Option D is true, and option E is not.

Linear regression gives $Y = aX + b$,

where $a = 641.586\,3992$

and $b = {}^{-}11.71013039$,

with the regression coefficient $r = {}^{-}0.9710135917$.

Exponential regression gives $Y = ab^X$,

where $a = 699.1256213$

and $b = 0.9730370621$,

with the regression coefficient $r = 0.9809837926$. Rounding a and b to 3 s.f. gives $Y = 699 \times (0.973)^X$. So option F is true.

Comparing regression coefficients suggests that power regression gives the best fit. So option A is true, and options B and C are not.

Putting the power regression function into $Y1$ and using either the Trace or Table facility gives $X = 15.764$ when $Y = 450$, and $X = 12.773$ when $Y = 500$. So option G is true, and option H is not.

Reference: *Calculator Book*, Chapters 10.1, 12.4, 13.1 and 13.2.

16

When $t = 0$, $y = 25 + 45 = 70$. So option H is correct.

As t increases, $\exp({}^{-}0.14t)$ tends to zero, and so y tends to 25. Hence option B is correct.

(Note: exp is entered using the e^x button on the calculator.)

Reference: *Calculator Book*, Chapter 12.3, *Unit 12*, Section 6, and *Unit 13*, Sections 4–6.

17

The ratio of the sides of the given triangle is $12:8:16$, or $3:2:4$ if you divide by 4 (or $2:3:4$, etc. — bear in mind that the order does not matter).

The sides in option B are in this ratio (divide by 2).

The sides in option E are in this ratio (multiply by 8).

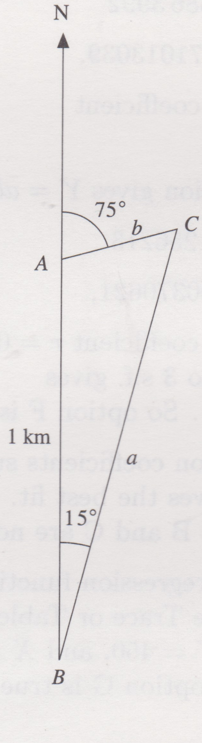
None of the other options have this ratio.

Options B and E are correct.

Reference: *Unit 14*, Section 2.

18

In the diagram below, A and B represent the points from which bearings are taken, and C represents the cairn.



In triangle ABC, angle $\widehat{A} = 105°$, $\widehat{B} = 15°$, so $\widehat{C} = 60°$. Application of the sine formula with the distances in kilometres ($AB = 1$) gives

$$\frac{a}{\sin 105°} = \frac{b}{\sin 15°} = \frac{1}{\sin 60°}.$$

So

$$a = \frac{\sin 105°}{\sin 60°} = 1.1153\ldots,$$

$$b = \frac{\sin 15°}{\sin 60°} = 0.2988\ldots.$$

These are distances in kilometres. Multiply by 1000 to get the distances in metres; these are 1115 and 299 (to the nearest whole number).

Options B and G are correct.

Reference: *Unit 14*, Section 3 and Appendix 3.

19

From the *Calculator Book*,

$LT1 = 13°45' = 13 + 45/60,$

$LG1 = 100°35' = 100 + 35/60,$

$LT2 = 40°45' = 40 + 45/60,$

$LG2 = {}^{-}74°.$

Use the calculator program GRTCIRC (make sure your calculator is in degree mode) and so find the great circle distance from Bangkok to New York = 13922.61374 $\simeq$ 14 000 km (to the nearest 1000 km).

Option E is correct.

Reference: *Unit 14*, Section 4, and *Calculator Book*, Chapter 14.2.

20

The distance on a terrestrial globe is given by θr, where θ is the subtended angle in radians and r is the radius of the globe. Now $100° = \frac{100}{180}\pi$ radians and $r = 1$ m, so distance on globe $= \frac{100}{180}\pi \simeq 1.7$ m (to 2 s.f.).

Option E is correct.

Reference: *Unit 14*, Section 4.

21

A sine wave $A\sin(wt + \phi)$ has amplitude A, period $2\pi/w$ and relative phase shift ϕ.

In this case, $A = 20$, $2\pi/w = 0.5$ (so $\omega = 4\pi$), and $\phi = \dfrac{45\pi}{180} = \dfrac{\pi}{4}$.

Therefore the given sine wave is represented by

$20\sin(4\pi t + \pi/4)$.

Option B is correct.

Reference: *Unit 15*, Section 2, and *Calculator Book*, Chapter 15.1.

22

Application of the procedure in the *Calculator Book*, Section 15.1, gives the time of sunrise, y, as $y = a\sin(bx + c) + d$, where $a = 2.56\ldots$, $b = 0.1158\ldots$, $c = 1.895\ldots$, $d = 6.22\ldots$.

So, with coefficients rounded to 2 s.f.,
$y = 2.6\sin(0.12x + 1.9) + 6.2$.

Option C is correct.

23

Try substituting some values of x into the options to find out which options are true. For each option, you might also plot the graphs corresponding to the functions on either side of the equals sign to see if they coincide.

A Not true (if $x = \pi/2$, for example).

B Not true (if $x = 0$, for example).

C True: the sine graph for negative values is ($^{-}1$) times that for positive values.

D Not true (if $x = 0$, for example).

E Not true (if $x = \pi/2$, for example).

F True: the cosine graph repeats every 2π.

G Not true (if $x = \pi/2$, for example).

H Not true (if $x = 0$, for example).

Reference: *Unit 15*, Section 2.

24

A Not true: the angle of the primary bow is about 42°.

B True

C True

D Not true: reflection is also involved.

E Not true: violet light is refracted more than red light.

F True

G True: the angle of the secondary bow is about 51°, which is 9° more than that of the primary bow.

H Not true

Reference: *Unit 16*, Section 2.

25

Use Appendix A of the calculator manual if you are unsure.

A Correct

B Incorrect: you cannot move up from 10 to 0 in steps of +2 (it is the wrong direction).

C Correct

D Correct

E Correct

F Incorrect: you cannot have 0.5 decimal places.

G Correct

H Incorrect: you can only input one variable at a time (unlike **Prompt**).

Reference: *Calculator Book*, Chapter 16.

26

The program requires the input of **K**, the weight of the joint, which it multiplies by 2.2. (and then by the time **T**). So the program converts kilograms to pounds before multiplying by the time per pound, **T**, which can be entered in any units.

Options A and G are true.

The brackets in the last line are not needed as **(K × 2.2) × T = K × 2.2 × T**.

So option F is true.

The program is based on the assumption that the cooking time is a constant multiplied by the weight.

So option H is also true.

Reference: *Calculator Book*, Chapter 16.

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